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A General Method of Estimation of Parameters with Known A Priori In Normal Parent

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ABSTRACT

This paper presents a general class of estimators for estimating various measures in normal parent when some a priori or guessed value σ_0 of standard deviation σ is available in addition to sample information. The bias and mean squared error (*MSE*) of the suggested class of estimators are obtained. Regions are identified under which proposed class of estimators are better than the usual unbiased estimator and the minimum mean square error (*MMSE*) estimator. Numerical computation in terms of percent relative efficiency established the superiority of the suggested class of estimators especially for small samples. The beauty of this paper lies in the unification of the problem of estimation of the various measures of normal parent such as standard deviation, variance, Fisher information, precision of sample mean, process capability index C_p, fourth moment about mean, mean deviation about mean etc.

Keywords: Normal parent, shrinkage estimators, guessed value, bias, mean squared error

I. INTRODUCTION

It is well known that the normal distribution plays a prominent role in both the theory and application of statistics. The various measures in normal parent such as mean, standard deviation, variance, Fisher information, precision of sample mean, process capability index, fourth moment about mean, mean deviation about mean and so on, assume importance in different situations of practical importance. These parameters have been estimated using different estimation procedure in isolation. Here we give a general procedure through which estimates of these parameters can be derived. All these measures are the function of the standard deviation of the form:

$$Q_{(\delta,\alpha)} = \delta\sigma^{\alpha}, \tag{1.1}$$

where δ is a positive real number and α is a non-zero finite integer. Putting the suitable values of the constants (δ,α) in (1.1) one can obtain the different measures.

Let $X_1, X_2, ..., X_n$ be a random sample of size *n* from a normal population $N(\mu,\sigma^2)$, probability density function of which is given by

$$f(x;\ \mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\},\quad -\infty < x < \infty, -\infty < \mu < \infty,\ \sigma > 0 \tag{1.2}$$

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where μ being the population mean acts as a location parameter and σ^2 being the population variance acts as a scale parameter.

An unbiased estimator of the parameter $Q_{(\delta,\alpha)}$ is defined by

$$\hat{Q}^{(u)}_{(\delta,\alpha)} = \delta C_{(n,\alpha)} s^{\alpha} \tag{1.3}$$

where

$$C_{(n,\alpha)} = \left(\frac{n-1}{2}\right)^{\alpha/2} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n+\alpha-1}{2}\right)},$$

$$s = \left[\sum_{i=1}^{n}(x_i-\bar{x})^2 \Big/ (n-1)\right]^{1/2}$$

and

$$\bar{x} = \sum_{i=1}^{n}\frac{x_i}{n} \text{ is the sample mean.}$$

The variance of $\hat{Q}^{(u)}_{(\delta,\alpha)}$ is given by

$$Var\left(\hat{Q}^{(u)}_{(\delta,\alpha)}\right) = \delta^2\left(G_{(n,\alpha)} - 1\right)\sigma^{2\alpha} \tag{1.4}$$

where

$$G_{(n,\alpha)} = \frac{\Gamma\left(\frac{n-1}{2}\right)\Gamma\left(\frac{n+2\alpha-1}{2}\right)}{\Gamma^2\left(\frac{n+\alpha-1}{2}\right)} \tag{1.5}$$

Following Searls (1964), Singh et.al. (1973) and Searls and Intarapanich (1990), we obtain a the minimum mean squared error (MMSE) estimator

$$\hat{Q}^{(m)}_{(\delta,\alpha)} = \delta\left(\frac{n-1}{2}\right)^{\alpha/2} \frac{\Gamma\left(\frac{n+\alpha-1}{2}\right)}{\Gamma\left(\frac{n+2\alpha-1}{2}\right)} s^{\alpha} \tag{1.6}$$

of $Q_{(\delta,\alpha)}$ in the class of estimators $M\hat{Q}^{(u)}_{(\delta,\alpha)}$, M being a suitably chosen scalar such that mean square error (MSE) of $M\hat{Q}^{(u)}_{(\delta,\alpha)}$ is minimum.

The bias and *MSE* of $\hat{Q}^{(m)}_{(\delta,\alpha)}$ are respectively given by

$$B\left(\hat{Q}^{(m)}_{(\delta,\alpha)}\right)=\delta\sigma^{\alpha}\frac{\left(1-G_{(n,\alpha)}\right)}{G_{(n,\alpha)}} \tag{1.7}$$

and

$$MSE\left(\hat{Q}^{(m)}_{(\delta,\alpha)}\right)=\frac{\delta^{2}\sigma^{2\alpha}\left(G_{(n,\alpha)}-1\right)}{G_{(n,\alpha)}} \tag{1.8}$$

Thompson (1968) was the first who initiated the problem of shrinking an estimator $\hat{\theta}$ (may be *MLE* or *UMVUE* or some other estimators) of the parameter θ towards a natural origin θ_0 and proposed a shrinkage estimator $\phi\hat{\theta}+(1-\phi)\theta_0$, where ϕ is a constant such that ϕ lies between 'zero' and 'one'. The beauty of such type of shrinkage estimators lies in the fact that, though perhaps they are biased, has smaller MSE than $\hat{\theta}$ of θ in some interval around θ_0 (the so called effective interval). The problem of shrinking the *MLE* $\hat{\theta}$ of θ towards a natural origin θ_0 was also studied by Mehta and Srinivasan (1971). Later several authors including Pandey and Singh (1977), Pandey (1979), Singh and Singh (1997), Singh and Saxena (2003a, 2003b), Saxena and Singh (2004) and Singh et al (2004) etc. have discussed the problem of estimating the population variance σ^2 of normal distribution in presence of guessed value σ_0^2 of σ^2.

Motivated by Jani (1991) and Kourouklis (1994), Singh and Singh (1997) suggested a class of estimators for population variance σ^2 as

$$\hat{Q}_{(1,2,p)}=\sigma_0^2\left[1+W_1\left(\frac{s^2}{\sigma_0^2}\right)^p\right] \tag{1.9}$$

where p is a non-zero real number and W_1 is a constant such that *MSE* of $\hat{Q}_{(1,2,p)}$ is minimum.

Singh et al (1999) considered a class of estimators of population standard deviation σ as

$$\hat{Q}_{(1,1,-p)}=\sigma_0\left\{1+W_1\left(\frac{s}{\sigma_0}\right)^{-p}\right\} \tag{1.10}$$

where W_1 and p are same as defined earlier.

Thus keeping in view (1.9) and (1.10), one can define a class of estimators of the parameter $Q_{(\delta,\alpha)}$ as

$$\hat{Q}_{(\delta,\alpha,p)} = \delta\sigma_0^{\alpha}\left[1+W_1\left(\frac{s^{\alpha}}{\sigma_0^{\alpha}}\right)^{p}\right] \tag{1.11}$$

where (δ,α,p,W_1) are same as defined earlier.

It is well known result that

$$E\left(s^{jk}\right) = K_{(j,k)}\sigma^{jk} \tag{1.12}$$

with

$$K_{(j,k)} = \left(\frac{2}{n-1}\right)^{\frac{jk}{2}} \frac{\Gamma\left(\frac{n+jk-1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)},$$

where (j,k) are non-zero real numbers, Gupta and Kapoor (2000). However, we note that the result (1.12) can be also obtained from equation (29.3.2) given in Cramer (1974).

The mean squared error of $\hat{Q}_{(\delta,\alpha,p)}$ is obtained as follows:

$$MSE\left(\hat{Q}_{(\delta,\alpha,p)}\right) = E\left[\delta^2\sigma^{2\alpha}r^2 + W_1^2\delta^2\sigma_0^{2\alpha(1-p)}s^{2\alpha p} + 2W_1\delta^2(\lambda-1)\sigma^{\alpha}\sigma_0^{\alpha(1-p)}s^{\alpha p}\right] \tag{1.13}$$

where

$$\lambda = \frac{\sigma_0^{\alpha}}{\sigma^{\alpha}} \text{ and } r = (\lambda - 1).$$

Using the result at (1.12) in (1.13) we get the *MSE* of $\hat{Q}_{(\delta,\alpha,p)}$ is given by

$$MSE\left(\hat{Q}_{(\delta,\alpha,p)}\right) = \delta^2\sigma^{2\alpha}\left[r^2 + W_1^2(1+r)^{2(1-p)}K_{(2\alpha,p)} + 2W_1r(1+r)^{(1-p)}K_{(\alpha,p)}\right] \tag{1.14}$$

which is minimum when

$$W_1 = -\frac{r}{(1+r)^{(1-p)}}K^{*}_{(\alpha,p)} \tag{1.15}$$

where

$$K^{*}_{(\alpha,p)} = \frac{K_{(\alpha,p)}}{K_{(2\alpha,p)}} = \left(\frac{n-1}{2}\right)^{\alpha p/2} \frac{\Gamma\left(\frac{n+\alpha p-1}{2}\right)}{\Gamma\left(\frac{n+2\alpha p-1}{2}\right)} \tag{1.16}$$

Since (1.15) contains unknown parameter σ^{α}, therefore replacing σ^{α} by its unbiased estimator $C_{(n,\alpha)}s^{\alpha}$, we get a consistent estimate of W_1 as

$$\hat{W}_1^{*} = -\left\{\frac{\sigma_0^{\alpha}}{C_{(n,\alpha)}s^{\alpha}} - 1\right\}\left\{\frac{\sigma_0^{\alpha}}{C_{(n,\alpha)}s^{\alpha}}\right\}^{(p-1)} K^{*}_{(\alpha,p)} \tag{1.17}$$

Substitution of (1.17) in (1.11) yields a class of shrinkage estimators of $Q_{(\delta,\alpha)}$ as

$$\hat{Q}^{*}_{(\delta,\alpha,p)} = \delta\sigma_0^{\alpha} + \delta W^{*}_{(\alpha,p)}\left(C_{(n,\alpha)}s^{\alpha} - \sigma_0^{\alpha}\right) \tag{1.18}$$

where

$$W^{*}_{(\alpha,p)} = C^{-p}_{(n,\alpha)} K^{*}_{(\alpha,p)}.$$

The expressions of bias and *MSE* of $\hat{Q}^{*}_{(\delta,\alpha,p)}$ are respectively given by

$$B\left\{\hat{Q}^{*}_{(\delta,\alpha,p)}\right\} = \delta\sigma^{\alpha}(\lambda - 1)\left\{1 - W^{*}_{(\alpha,p)}\right\} \tag{1.19}$$

and

$$MSE\left\{\hat{Q}^{*}_{(\delta,\alpha,p)}\right\} = \delta^2\sigma^{2\alpha}\left[(\lambda-1)^2\left\{1 - W^{*}_{(\alpha,p)}\right\}^2 + \left\{G_{(n,\alpha)} - 1\right\}W^{*2}_{(\alpha,p)}\right] \tag{1.20}$$

In this paper we have suggested a class of estimators of $Q_{(\delta,\alpha)}$ using the same technique as adopted by Singh et al. (1993) and Singh and Saxena (2003a) when a guessed value σ_0 of the population standard deviation σ is available. The properties of the suggested class of estimators are studied theoretically and empirically.

II. THE PROPOSED CLASS OF SHRINKAGE ESTIMATORS

Suppose σ_0 is a prior point estimate or guess value of the population standard deviation σ obtained from past studies or experience. Then we define a class of estimators for estimating the parameter $Q_{(\delta,\alpha)} = \delta\sigma^{\alpha}$ as

$$T_{(\delta,\alpha,p,q)} = \delta\sigma_0^{\alpha}\left[1 + W_1\left(\frac{s^{\alpha}}{\sigma_0^{\alpha}}\right)^p + W_2\left(\frac{\sigma_0^{\alpha}}{s^{\alpha}}\right)^q\right], \tag{2.1}$$

where p and q are non-zero real numbers such that $p+q \neq 0$ and W_i's (i=1,2) are constants to be determined such that *MSE* of $T_{(\delta,\alpha,p,q)}$ is minimum.

The *MSE* of $T_{(\delta,\alpha,p,q)}$ is given by

$$MSE\left(T_{(\delta,\alpha,p,q)}\right)=\delta^2\sigma^{2\alpha}\Big[r^2+W_1^2(1+r)^{2(1-p)}K_{(2\alpha,p)}+W_2^2(1+r)^{2(1+q)}K_{(-2\alpha,q)}$$
$$+2W_1W_2(1+r)^{(2-p+q)}K_{(\alpha,p-q)}+2W_1r(1+r)^{(1-p)}K_{(\alpha,p)}$$
$$+2W_2r(1+r)^{(1+q)}K_{(-\alpha,q)}\Big] \tag{2.2}$$

Minimization of (2.2) with respect to W_1 and W_2 yield the following normal equation:

$$\begin{bmatrix}(1+r)^{2(1-p)}K_{(2\alpha,p)} & K_{(\alpha,p-q)}(1+r)^{(2-p+q)}\\ K_{(\alpha,p-q)}(1+r)^{(2-p+q)} & (1+r)^{2(1+q)}K_{(-2\alpha,q)}\end{bmatrix}\begin{bmatrix}W_1\\ W_2\end{bmatrix}=\begin{bmatrix}-r(1+r)^{(1-p)}K_{(\alpha,p)}\\ -r(1+r)^{(1+q)}K_{(-\alpha,q)}\end{bmatrix} \tag{2.3}$$

Solving (2.3) we get the optimum values of W_1 and W_2 as

$$W_1=\frac{-r\left[K_{(\alpha,p)}K_{(-2\alpha,q)}-K_{(\alpha,p-q)}K_{(-\alpha,q)}\right]}{(1+r)^{(1-p)}\left[K_{(2\alpha,p)}K_{(-2\alpha,q)}-K^2_{(\alpha,p-q)}\right]} \tag{2.4}$$

$$W_2=\frac{-r\left[K_{(2\alpha,p)}K_{(-\alpha,q)}-K_{(\alpha,p)}K_{(\alpha,p-q)}\right]}{(1+r)^{(1+q)}\left[K_{(2\alpha,p)}K_{(-2\alpha,q)}-K^2_{(\alpha,p-q)}\right]} \tag{2.5}$$

Since σ^α is not known in (2.4) and (2.5), therefore replacing σ^α by its unbiased estimator $C_{(n,\alpha)}s^\alpha$, we get the estimates of W_1 and W_2 respectively as

$$\hat{W}_1=-\left\{\frac{\sigma_0^\alpha}{C_{(n,\alpha)}s^\alpha}-1\right\}\left\{\frac{\sigma_0^\alpha}{C_{(n,\alpha)}s^\alpha}\right\}^{(p-1)}W_1'(\alpha,p,q), \tag{2.6}$$

$$\hat{W}_2=-\left\{\frac{\sigma_0^\alpha}{C_{(n,\alpha)}s^\alpha}-1\right\}\left\{\frac{\sigma_0^\alpha}{C_{(n,\alpha)}s^\alpha}\right\}^{-(q+1)}W_2'(\alpha,p,q), \tag{2.7}$$

where

$$W_1'(\alpha,p,q)=\frac{\left[K_{(\alpha,p)}K_{(-2\alpha,q)}-K_{(-\alpha,q)}K_{(\alpha,p-q)}\right]}{\left[K_{(2\alpha,p)}K_{(-2\alpha,q)}-K^2_{(\alpha,p-q)}\right]}, \tag{2.8}$$

$$W_2'(\alpha,p,q)=\frac{\left[K_{(2\alpha,p)}K_{(-\alpha,q)}-K_{(\alpha,p)}K_{(\alpha,p-q)}\right]}{\left[K_{(2\alpha,p)}K_{(-2\alpha,q)}-K^2_{(\alpha,p-q)}\right]}. \tag{2.9}$$

Substitution of (2.8) and (2.9) in (2.1) yields a class of shrinkage estimators of $Q_{(\delta,\alpha)}$ as

$$\hat{T}_{(\delta,\alpha,p,q)}=\delta\sigma_0^\alpha+\delta W(\alpha,p,q)\left(C_{(n,\alpha)}s^\alpha-\sigma_0^\alpha\right)$$

or

$$\hat{T}_{(\delta,\alpha,p,q)} = \delta\sigma_0^\alpha\{1-W(\alpha,p,q)\}+W(\alpha,p,q)\hat{Q}_{(\delta,\alpha)}^{(u)}, \tag{2.10}$$

where

$$W(\alpha,p,q)=\left[\frac{W_1'(\alpha,p,q)}{C_{(n,\alpha)}^p}+C_{(n,\alpha)}^q W_2'(\alpha,p,q)\right] \tag{2.11}$$

We note that for different values of δ and α, the derived class of shrinkage estimators in (2.10) reduces to a class of estimators of certain parametric functions. Some of these are listed below:

Table 1 - Classes of estimators of different parametric functions

Parametric Function	Values of (δ,α)	Class of Shrinkage Estimators
1.Population Standard Deviation (σ)	(1,1)	$\hat{T}_{(1,1,p,q)} = \sigma_0\{1-W(1,p,q)\}+W(1,p,q)\hat{Q}_{(1,1)}^{(u)}$
2.Population Variance (σ^2)	(1,2)	$\hat{T}_{(1,2,p,q)} = \sigma_0^2\{1-W(2,p,q)\}+W(2,p,q)\hat{Q}_{(1,2)}^{(u)}$ which is due to Singh and Saxena (2003a).
3.Fisher Information $\left(\frac{1}{\sigma^2}\right)$	(1,-2)	$\hat{T}_{(1,-2,p,q)} = \frac{1}{\sigma_0^2}\{1-W(-2,p,q)\}+W(-2,p,q)\hat{Q}_{(1,-2)}^{(u)}$
4.Precision of sample mean $\left(\frac{n}{\sigma^2}\right)$	(*n*,-2)	$\hat{T}_{(n,-2,p,q)} = \frac{n}{\sigma_0^2}\{1-W(-2,p,q)\}+W(-2,p,q)\hat{Q}_{(n,-2)}^{(u)}$
5.Mean deviation about mean $\left(\sigma\sqrt{\frac{2}{\pi}}\right)$	$\left(\sqrt{\frac{2}{\pi}},1\right)$	$\hat{T}_{\left(\sqrt{\frac{2}{\pi}},1,p,q\right)} = \sqrt{\frac{2}{\pi}}\{1-W(1,p,q)\}\sigma_0+W(1,p,q)\hat{Q}_{\left(\sqrt{\frac{2}{\pi}},1\right)}^{(u)}$
6.Fourth moment about mean $(3\sigma^4)$	(3,4)	$\hat{T}_{(3,4,p,q)} = 3\sigma_0^4\{1-W(4,p,q)\}+W(4,p,q)\hat{Q}_{(3,4)}^{(u)}$
7.Process Capability Index $\left(C_p=\frac{d}{6\sigma}\right)$ where d = USL-LSL, USL=Upper specification limit LSL=Lower specification limit	$\left(\frac{d}{6},-1\right)$	$\hat{T}_{\left(\frac{d}{6},-1,p,q\right)} = \frac{d}{6\sigma_0}\{1-W(-1,p,q)\}+W(-1,p,q)\hat{Q}_{\left(\frac{d}{6},-1\right)}^{(u)}$

We note from (2.10) that the estimator $\hat{T}_{(\delta,\alpha,p,q)}$ is a convex combination of $\delta\sigma_0^\alpha$ and $\hat{Q}_{(\delta,\alpha)}^{(u)}$ if $W(\alpha,p,q)>0$ and $\{1-W(\alpha,p,q)\}>0$ *i.e.* $0<W(\alpha,p,q)\le 1$. Since we are

interested in estimating a non-negative parameter $Q_{(\delta,\alpha)}$, therefore, it is necessary that the estimator $\hat{T}_{(\delta,\alpha,p,q)}$ should be non-negative $\left(i.e.\hat{T}_{(\delta,\alpha,p,q)} > 0\right)$. This leads to impose a condition:

$$0 < W(\alpha, p, q) \le 1 \tag{2.12}$$

Therefore acceptable range of values of (*p, q*) for all n and α is given by

$$\{(p,q) | 0 < W(\alpha, p, q) \le 1\} \tag{2.13}$$

If $W(\alpha, p, q) = 1$, the suggested class of estimators turns into the unbiased estimator $\hat{Q}^{(u)}_{(\delta,\alpha)}$, otherwise it is biased with

$$B\{\hat{T}_{(\delta,\alpha,p,q)}\} = \delta\sigma^{\alpha}(\lambda - 1)\{1 - W(\alpha, p, q)\} \tag{2.14}$$

The *MSE* of $\hat{T}_{(\delta,\alpha,p,q)}$ is given by

$$MSE\{\hat{T}_{(\delta,\alpha,p,q)}\} = \delta^2\sigma^{2\alpha}\left[(\lambda - 1)^2\{1 - W(\alpha, p, q)\}^2 + \left(G_{(n,\alpha)} - 1\right)W^2(\alpha, p, q)\right] \tag{2.15}$$

It follows from (2.14) that $B\{\hat{T}_{(\delta,\alpha,p,q)}\}$ is zero if $\lambda = 1$ i.e. if the guessed value σ_0^{α} coincides exactly with the true value σ^{α}. It is also evident from (2.15) that the $MSE\{\hat{T}_{(\delta,\alpha,p,q)}\}$ is minimum when $\lambda = 1$ or $\sigma_0^{\alpha} = \sigma^{\alpha}$. Thus for $\lambda = 1$, the minimum *MSE* of $\hat{T}_{(\delta,\alpha,p,q)}$ is given by

$$\text{min.}\, MSE\{\hat{T}_{(\delta,\alpha,p,q)}\} = \left(G_{(n,\alpha)} - 1\right)W^2(\alpha, p, q)\delta^2\sigma^{2\alpha}. \tag{2.16}$$

The quantity $\lambda = \sigma_0^{\alpha} / \sigma^{\alpha}$ indicates the departure of natural origin σ_0^{α} from the true value σ^{α}. However in some practical situations, it is difficult to have the exact value of λ. To overcome such difficulty an unbiased estimator of λ is propounded as a guideline to know in practice whether λ is within its acceptable range of dominance or not. An unbiased estimator of λ is given by

$$\hat{\lambda} = \left(\frac{2}{n-1}\right)^{\alpha/2} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-\alpha-1}{2}\right)} \left(\frac{\sigma_0}{s}\right)^{\alpha} \tag{2.17}$$

with the variance

$$V(\hat{\lambda}) = \lambda^2 \left[\frac{\Gamma\left(\frac{n-1}{2}\right)\Gamma\left(\frac{n-2\alpha-1}{2}\right)}{\Gamma^2\left(\frac{n-\alpha-1}{2}\right)} - 1\right] \tag{2.18}$$

III. EFFICIENCY COMPARISON OF ESTIMATORS

James and Stein (1961) have mentioned that the minimum *MSE* is highly desirable property and it is therefore used as a criterion to compare various estimators with each other.

It follows from (1.4) and (2.15) that the proposed estimator $\hat{T}_{(\delta,\alpha,p,q)}$ is more efficient than the unbiased estimator $\hat{Q}^{(u)}_{(\delta,\alpha)}$ if

$$\left(1-\sqrt{A_1}\right)<\lambda<\left(1+\sqrt{A_1}\right), \tag{3.1}$$

where

$$A_1=\left[\frac{\{1+W(\alpha,p,q)\}\{G_{(n,\alpha)}-1\}}{\{1-W(\alpha,p,q)\}}\right]. \tag{3.2}$$

It is observed from (1.8) and (2.15) that

$$MSE\left\{\hat{T}_{(\delta,\alpha,p,q)}\right\}<MSE\left\{\hat{Q}^{(m)}_{(\delta,\alpha)}\right\} \text{ if}$$

$$\left[(\lambda-1)^2\{1-W(\alpha,p,q)\}^2+\left(G_{(n,\alpha)}-1\right)W^2(\alpha,p,q)\right]<\frac{\left(G_{(n,\alpha)}-1\right)}{G_{(n,\alpha)}}$$

i.e. if

$$\left(1-\sqrt{A_2}\right)<\lambda<\left(1+\sqrt{A_2}\right), \tag{3.3}$$

where

$$A_2=\left[\frac{\{1-W^2(\alpha,p,q)G_{(n,\alpha)}\}\{G_{(n,\alpha)}-1\}}{\{1-W(\alpha,p,q)\}^2G_{(n,\alpha)}}\right].$$

From (1.20) and (2.15) we note that

$$MSE\left\{\hat{T}_{(\delta,\alpha,p,q)}\right\}<MSE\left\{\hat{Q}^{*}_{(\delta,\alpha,p)}\right\} \text{ if}$$

$$\left[(\lambda-1)^2\left(2-W(\alpha,p,q)-W^*(\alpha,p,q)\right)\left(W^*(\alpha,p,q)-W(\alpha,p,q)\right)\right]$$
$$<\left[\left(G_{(n,\alpha)}-1\right)\left(W(\alpha,p,q)+W^*(\alpha,p,q)\right)\left(W^*(\alpha,p,q)-W(\alpha,p,q)\right)\right]$$

i.e. if

either $\left(1-\sqrt{A_3}\right)<\lambda<\left(1+\sqrt{A_3}\right),\ W^*(\alpha,p,q)>W(\alpha,p,q)$ (3.4)

or $\lambda\notin\left(1-\sqrt{A_3},1+\sqrt{A_3}\right),\ W^*(\alpha,p,q)<W(\alpha,p,q),$ (3.5)

where

$$A_3 = \frac{\left(G_{(n,\alpha)}-1\right)\left\{W(\alpha,p,q)+W^*(\alpha,p,q)\right\}}{\left\{2-W^*(\alpha,p,q)-W(\alpha,p,q)\right\}} \tag{3.6}$$

In addition to minimum *MSE* criterion, minimum bias is also of great consequence criterion and therefore should be taken under consideration. Thus absolute relative bias (*ARB*) of the suggested estimator $\hat{T}_{(\delta,\alpha,p,q)}$ is less than that of $\hat{Q}^{(m)}_{(\delta,\alpha)}$ if

$$\left|(\lambda-1)\{1-W(\alpha,p,q)\}\right| < \left|\frac{\left(1-G_{(n,\alpha)}\right)}{G_{(n,\alpha)}}\right|$$

i.e. if

$$\left(1-A_4\right) < \lambda < \left(1+A_4\right), \tag{3.7}$$

where

$$A_4 = \frac{\left(1-G_{(n,\alpha)}\right)}{G_{(n,\alpha)}\{1-W(\alpha,p,q)\}}$$

It is observed from (1.19) and (2.14) that the *ARB* of $\hat{T}_{(\delta,\alpha,p,q)}$ is smaller than that of $\hat{Q}^*_{(\delta,\alpha,p)}$ if

$$\{1-W(\alpha,p,q)\}^2 < \left\{1-W^*(\alpha,p,q)\right\}^2 \tag{3.8}$$

IV. NUMERICAL ILLUSTRATION

To get a perceptible idea about the performance of various estimators of $Q_{(\delta,\alpha)}$ relative to *MMSE* estimator $\hat{Q}^{(m)}_{(\delta,\alpha)}$ and the estimator $\hat{Q}^*_{(\delta,\alpha,p)}$, we have computed the percent relative efficiencies (*PREs)* of $\hat{T}_{(\delta,\alpha,p,q)}$ with respect to *MMSE* $\hat{Q}^{(m)}_{(\delta,\alpha)}$ and the estimator $\hat{Q}^*_{(\delta,\alpha,p)}$, by using the formula:

$$PRE\left\{\hat{T}_{(\delta,\alpha,p,q)},\hat{Q}^{(m)}_{(\delta,\alpha)}\right\} = \frac{\left\{G_{(n,\alpha)}-1\right\}}{G_{(n,\alpha)}\left[(\lambda-1)^2\{1-W(\alpha,p,q)\}^2+\left(G_{(n,\alpha)}-1\right)W^2(\alpha,p,q)\right]}\times 100 \tag{4.1}$$

and

$$PRE\left\{\hat{T}_{(\delta,\alpha,p,q)},\hat{Q}^{*}_{(\delta,\alpha,p)}\right\} = \frac{\left|(\lambda-1)^2\{1-W^*(\alpha,p,q)\}^2+\left(G_{(n,\alpha)}-1\right)W^*(\alpha,p)\right|}{\left[(\lambda-1)^2\{1-W(\alpha,p,q)\}^2+\left(G_{(n,\alpha)}-1\right)W^2(\alpha,p,q)\right]}\times 100 \tag{4.2}$$

It is observed from (4.1) and (4.2) that the above formulae are independent of δ and therefore to observe the merits of the proposed class of estimators in estimating different parametric functions the value of α is enough no matter what is the value of δ.

We have computed the $PRE\left(\hat{T}_{(\delta,\alpha,p,q)},\hat{Q}^{(m)}_{(\delta,\alpha)}\right)$ for the parametric combinations:

(1) α=-2; λ=0.05, 0.25 (0.25) 2.00; n=5(5) 20, p=1, 2; and q=1.25(0.50) 3.00;
(2) α =-1; λ=0.05, 0.25 (0.25) 2.00; n=5(5) 20, p=1, 2; and q=1.50(0.50) 3.00;
(3) α =1; λ=0.05, 0.25 (0.25) 2.00; n=5(5) 20, p=1, 2; and q=1.25(0.25) 2.00;

and $PRE\left(\hat{T}_{(\delta,\alpha,p,q)},\hat{Q}^{*}_{(\delta,\alpha,p)}\right)$ for
(1) α=-2; λ=0.05, 0.50 (0.50) 4.00; n=5(5) 20, p=1, 2; and q=1.25(0.50) 3.00;
(2) α=-1; λ=0.05, 0.50 (0.50) 4.00; n=5(5) 20, p=1, 2; and q=1.50(0.50) 3.00;
(3) α=1; λ=0.05, 0.50 (0.50) 4.00; n=5(5) 20, p=1,2; and q=1.25(0.25) 2.00.

The findings of *PREs* are listed in Tables 2 to 4 and 5 to 7 respectively.

Table 2. ***PRE*** **of** $\hat{T}_{(\delta,\alpha,p,q)}$ **with respect to** ***MMSE*** $\hat{Q}^{(m)}_{(\delta,\alpha)}$ **for** α**=-2 (Applicable for estimation of Fisher information and Precision of sample mean)**

q	λ	p=1				p=2			
		5	10	15	20	5	10	15	20
1.25	**0.05**	99.74	97.53	98.50	98.91	100.79	94.55	98.17	99.51
	0.25	115.70	102.12	100.99	100.61	145.95	121.71	112.94	108.92
	0.50	135.66	106.73	103.40	102.23	248.16	165.36	131.06	119.28
	0.75	151.32	109.70	104.90	103.23	427.99	210.70	145.03	126.50
	1.00	157.38	110.73	105.41	103.56	564.31	231.89	150.37	129.11
	1.25	151.32	109.70	104.90	103.23	427.99	210.70	145.03	126.50
	1.50	135.66	106.73	103.40	102.23	248.16	165.36	131.06	119.28
	1.75	115.70	102.12	100.99	100.61	145.95	121.71	112.94	108.92
	2.00	95.94	96.29	97.81	98.43	92.57	88.87	94.62	97.10
Range of λ		(0.53, 1.95)	(0.15, 1.85)	(0.16, 1.83)	(0.17, 1.83)	(0.04, 1.95)	(0.09, 1.90)	(0.07, 1.92)	(0.06, 1.93)
1.75	**0.05**	105.29	101.69	100.98	100.69	85.10	70.02	82.03	89.22
	0.25	130.66	110.22	105.66	103.92	133.23	103.63	106.51	106.66
	0.50	167.83	119.42	110.37	107.07	277.45	185.43	146.76	130.01
	0.75	202.38	125.71	113.40	109.05	791.68	352.23	189.79	149.67
	1.00	217.29	127.96	114.45	109.73	2071.32	503.09	210.35	157.62
	1.25	202.38	125.71	113.40	109.05	791.68	352.23	189.79	149.67
	1.50	167.83	119.42	110.37	107.07	277.45	185.43	146.76	130.01
	1.75	130.66	110.22	105.66	103.92	133.23	103.63	106.51	106.66
	2.00	99.74	99.49	99.71	99.80	77.11	64.07	76.96	85.23
Range of λ		(0.00, 1.99)	(0.01, 1.98)	(0.01, 1.98)	(0.01, 1.98)	(0.13, 1.87)	(0.23, 1.76)	(0.20, 1.79)	(0.17, 1.82)
2.25	**0.05**	106.15	102.89	101.74	101.25	77.47	53.32	65.47	75.95
	0.25	135.10	114.40	108.42	106.00	123.24	83.23	92.61	98.34
	0.50	180.32	127.53	115.37	110.78	269.67	171.85	149.65	134.90
	0.75	225.62	136.95	119.99	113.86	939.37	475.74	237.35	173.64
	1.00	246.24	140.41	121.61	114.93	5455.86	1158.82	294.97	192.02
	1.25	225.62	136.95	119.99	113.86	939.37	475.74	237.35	173.64
	1.50	180.32	127.53	115.37	110.78	269.67	171.85	149.65	134.90
	1.75	135.10	114.40	108.42	106.00	123.24	83.23	92.61	98.34
	2.00	100.00	100.00	99.98	99.96	70.01	48.34	60.39	71.29
Range of λ		(3.72E-06, 1.99)	(1.76E-05, 1.99)	(0.00, 1.99)	(0.00, 1.99)	(0.16, 1.83)	(0.32, 1.67)	(0.29, 1.70)	(0.26, 1.73)
3.00	**0.05**	106.24	103.08	101.73	101.07	74.19	42.23	50.23	60.59
	0.25	135.91	116.44	110.13	107.55	118.51	67.27	75.54	84.69
	0.50	182.84	132.19	119.19	114.28	262.83	147.94	140.74	133.50
	0.75	230.63	143.86	125.37	118.74	975.93	527.31	291.90	204.05
	1.00	252.63	148.23	127.58	120.31	10209.31	3630.77	454.66	247.67
	1.25	230.63	143.86	125.37	118.74	975.93	527.31	291.90	204.05
	1.50	182.84	132.19	119.19	114.28	262.83	147.94	140.74	133.50
	1.75	135.91	116.44	110.13	107.55	118.51	67.27	75.54	84.69
	2.00	99.98	99.79	99.55	99.36	67.00	38.15	45.83	56.02
Range of λ		(0.00, 1.99)	(0.00, 1.99)	(0.01, 1.98)	(0.02, 1.98)	(0.18, 1.81)	(0.38, 1.61)	(0.36, 1.63)	(0.35, 1.66)

Table 3. ***PRE*** **of** $\hat{T}_{(\delta,\alpha,p,q)}$ **with respect to** ***MMSE*** $\hat{Q}^{(m)}_{(\delta,\alpha)}$ **for** α**=-1**

(Applicable for estimation of process capability index C_p**)**

q	λ	$p=1$				$p=2$			
		5	10	15	20	5	10	15	20
1.50	**0.05**	99.52	99.88	99.95	99.97	97.62	98.69	99.50	99.73
	0.25	103.49	101.19	100.72	100.52	115.71	105.35	103.13	102.20
	0.50	107.44	102.42	101.45	101.03	139.47	112.31	106.70	104.59
	0.75	109.95	103.17	101.89	101.34	159.07	116.95	108.97	106.07
	1.00	110.81	103.42	102.03	101.45	166.89	118.59	109.74	106.57
	1.25	109.95	103.17	101.89	101.34	159.07	116.95	108.97	106.07
	1.50	107.44	102.42	101.45	101.03	139.47	112.31	106.70	104.59
	1.75	103.49	101.19	100.72	100.52	115.71	105.35	103.13	102.20
	2.00	98.44	99.51	99.73	99.81	93.43	96.93	98.51	99.04
Range of λ		(0.07 1.92)	(0.06, 1.94)	(0.06, 1.93)	(0.06, 1.93)	(0.07, 1.92)	(0.08, 1.91)	(0.07, 1.92)	(0.07, 1.92)
2.00	**0.05**	101.65	100.61	100.38	100.27	84.19	92.28	96.21	97.57
	0.25	108.29	102.85	101.73	101.24	110.56	103.76	102.65	101.99
	0.50	115.20	105.00	103.00	102.14	155.25	117.16	109.38	106.42
	0.75	119.78	106.34	103.77	102.69	204.96	127.00	113.86	109.26
	1.00	121.39	106.79	104.04	102.87	229.45	130.66	115.43	110.25
	1.25	119.78	106.34	103.77	102.69	204.96	127.00	113.86	109.26
	1.50	115.20	105.00	103.00	102.14	155.25	117.16	109.38	106.42
	1.75	108.29	102.85	101.73	101.24	110.56	103.76	102.65	101.99
	2.00	99.90	99.99	100.00	100.00	78.80	89.44	94.51	96.37
Range of λ		(0.00, 1.99)	90.00, 1.99)	(0.00, 1.99)	(0.00, 1.99)	(0.17, 1.82)	(0.18, 1.81)	(0.16, 1.83)	(0.15 1.84)
2.50	**0.05**	102.18	100.73	100.42	100.29	70.81	83.63	91.19	94.11
	0.25	111.05	103.96	102.41	101.74	99.91	99.28	100.61	100.74
	0.50	120.69	107.11	104.32	103.10	160.56	119.91	111.18	107.71
	0.75	127.32	109.10	105.49	103.94	252.55	136.99	118.65	112.38
	1.00	129.69	109.78	105.89	104.22	312.16	143.81	121.37	114.02
	1.25	127.32	109.10	105.49	103.94	252.55	136.99	118.65	112.38
	1.50	120.69	107.11	104.32	103.10	160.56	119.91	111.18	107.71
	1.75	111.05	103.96	102.41	101.74	99.91	99.28	100.61	100.74
	2.00	99.89	99.84	99.86	99.88	65.35	80.01	88.80	92.37
Range of λ		(0.00, 1.99)	(0.00, 1.99)	(0.01, 1.98)	(0.01, 1.98)	(0.25, 1.74)	(0.25, 1.74)	(0.23, 1.76)	(0.22, 1.77)
3.00	**0.05**	102.09	100.48	100.18	100.08	61.08	74.95	85.36	89.84
	0.25	112.45	104.62	102.83	102.04	89.89	93.37	97.46	98.66
	0.50	124.01	108.73	105.39	103.91	158.67	120.64	112.07	108.45
	0.75	132.16	111.36	106.99	105.07	293.37	146.26	123.15	115.31
	1.00	135.12	112.26	107.54	105.46	409.13	157.40	127.35	117.80
	1.25	132.16	111.36	106.99	105.07	293.37	146.26	123.15	115.31
	1.50	124.01	108.73	105.39	103.91	158.67	120.64	112.07	108.45
	1.75	112.45	104.62	102.83	102.04	89.89	93.37	97.46	98.66
	2.00	99.46	99.35	99.45	99.53	55.94	70.93	82.42	87.60
Range of λ		90.01, 1.98)	(0.02, 1.97)	(0.03, 1.96)	(0.04, 1.95)	(0.30, 1.69)	(0.31, 1.68)	(0.29, 1.70)	(0.28, 1.71)

Table 4. *PRE* of $\hat{T}_{(\delta,\alpha,p,q)}$ with respect to *MMSE* $\hat{Q}^{(m)}_{(\delta,\alpha)}$ for α=1 (Applicable for estimation of standard deviation (σ) and Mean deviation about mean)

q	λ	p=1				p=2			
		5	10	15	20	5	10	15	20
1.25	**0.05**	100.10	99.72	99.75	99.79	93.19	99.54	100.02	100.10
	0.25	102.64	100.61	100.29	100.17	108.27	104.10	102.53	101.81
	0.50	105.10	101.44	100.78	100.53	127.20	108.68	104.95	103.44
	0.75	106.63	101.95	101.08	100.74	142.10	111.63	106.45	104.44
	1.00	107.15	102.12	101.18	100.81	147.87	112.65	106.96	104.78
	1.25	106.63	101.95	101.08	100.74	142.10	111.63	106.45	104.44
	1.50	105.10	101.44	100.78	100.53	127.20	108.68	104.95	103.44
	1.75	102.64	100.61	100.29	100.17	108.27	104.10	102.53	101.81
	2.00	99.39	99.47	99.60	99.69	89.61	98.31	99.33	99.61
Range of λ		(0.04, 1.95)	(0.10, 1.89)	(0.13, 1.86)	(0.15, 1.84)	(0.14, 1.85)	(0.06, 1.93)	(0.04, 1.95)	(0.03, 1.96)
1.50	**0.05**	100.77	100.16	100.06	100.03	86.16	97.45	99.07	99.51
	0.25	104.14	101.41	100.81	100.57	105.11	104.07	102.73	102.01
	0.50	107.43	102.59	101.52	101.07	131.73	111.01	106.33	104.42
	0.75	109.51	103.30	101.95	101.38	155.34	115.63	108.62	105.92
	1.00	110.22	103.54	102.09	101.48	165.20	117.26	109.40	106.43
	1.25	109.51	103.30	101.95	101.38	155.34	115.63	108.62	105.92
	1.50	107.43	102.59	101.52	101.07	131.73	111.01	106.33	104.42
	1.75	104.14	101.41	100.81	100.57	105.11	104.07	102.73	102.01
	2.00	99.85	99.81	99.84	99.87	81.93	95.70	98.07	98.82
Range of λ		(0.01 1.99)	(0.02 1.96)	(0.03 1.95)	(0.04 1.94)	(0.19, 1.80)	(0.12, 1.87)	(0.09, 1.90)	(0.08, 1.91)
1.75	**0.05**	101.11	100.43	100.26	100.18	79.52	94.41	97.57	98.55
	0.25	105.20	102.05	101.26	100.90	100.95	103.24	102.52	101.95
	0.50	109.26	103.59	102.20	101.58	134.20	112.94	107.53	105.28
	0.75	111.85	104.54	102.77	101.99	167.25	119.68	110.78	107.39
	1.00	112.74	104.86	102.96	102.13	182.21	122.12	111.91	108.11
	1.25	111.85	104.54	102.77	101.99	167.25	119.68	110.78	107.39
	1.50	109.26	103.59	102.20	101.58	134.20	112.94	107.53	105.28
	1.75	105.20	102.05	101.26	100.90	100.95	103.24	102.52	101.95
	2.00	99.99	99.97	99.97	99.97	74.96	92.16	96.24	97.62
Range of λ		(0.00 1.99)	(0.00 1.99)	(0.00 1.99)	(0.01 1.99)	(0.24, 1.75)	(0.17 1.82)	(0.14, 1.85)	(0.13, 1.86)
2.00	**0.05**	101.24	100.55	100.36	100.26	74.02	90.71	95.60	97.25
	0.25	105.93	102.55	101.62	101.18	96.86	101.69	101.92	101.63
	0.50	110.64	104.45	102.81	102.05	135.20	114.41	108.53	106.01
	0.75	113.67	105.63	103.54	102.57	177.31	123.70	112.92	108.83
	1.00	114.72	106.03	103.78	102.75	197.85	127.13	114.46	109.81
	1.25	113.67	105.63	103.54	102.57	177.31	123.70	112.92	108.83
	1.50	110.64	104.45	102.81	102.05	135.20	114.41	108.53	106.01
	1.75	105.93	102.55	101.62	101.18	96.86	101.69	101.92	101.63
	2.00	99.97	100.00	100.00	100.00	69.33	87.99	93.92	96.06
Range of λ		(0.00 1.99)	(0.00 1.99)	(0.00 1.99)	(0.00 1.99)	(0.27, 1.72)	(0.21, 1.78)	(0.18, 1.81)	(0.17, 1.82)

Table 5. ***PRE*** **of $\hat{T}_{(\delta,\alpha,p,q)}$ with respect to $\hat{Q}^{*}_{(\delta,\alpha,p)}$ for α=-2 (Applicable for estimation of Fisher information and Precision of sample mean)**

q	λ	p=1				p=2			
		5	10	15	20	5	10	15	20
1.25	**0.05**	153.01	122.62	113.31	109.41	150.67	265.68	275.52	250.61
	0.50	155.73	114.33	107.71	105.24	128.58	141.44	139.80	131.58
	1.00	157.38	110.73	105.41	103.56	81.18	24.69	60.15	72.40
	1.50	155.73	114.33	107.71	105.24	128.58	141.44	139.80	131.58
	2.00	152.72	123.73	114.11	110.01	151.90	275.66	290.16	265.09
	2.50	150.34	135.89	123.36	117.22	159.00	343.55	413.77	403.39
	3.00	148.80	148.12	134.02	126.01	161.83	376.89	495.70	514.42
	3.50	147.83	158.98	144.85	135.54	163.21	394.81	548.02	595.67
	4.00	147.21	168.02	155.06	145.11	163.98	405.33	582.08	653.71
1.75	**0.05**	161.53	127.84	116.17	111.38	127.22	196.76	230.23	224.71
	0.50	192.67	127.92	114.97	110.22	143.75	158.60	156.54	143.41
	1.00	217.29	127.96	114.45	109.73	297.98	53.57	84.14	88.39
	1.50	192.67	127.92	114.97	110.22	143.75	158.60	156.54	143.41
	2.00	158.76	127.83	116.33	111.54	126.53	198.73	236.01	232.67
	2.50	138.73	127.73	118.10	113.39	122.91	209.78	274.75	294.63
	3.00	128.04	127.65	119.89	115.43	121.61	214.06	293.44	331.11
	3.50	122.04	127.59	121.48	117.41	121.00	216.12	303.39	352.75
	4.00	118.43	127.54	122.81	119.19	120.66	217.26	309.20	366.21
2.25	**0.05**	162.84	129.34	117.05	112.00	115.81	149.82	183.75	191.27
	0.50	207.00	136.61	120.18	114.04	139.72	146.99	159.63	148.81
	1.00	246.24	140.41	121.61	114.93	784.87	123.39	117.99	107.68
	1.50	207.00	136.61	120.18	114.04	139.72	146.99	159.63	148.81
	2.00	159.18	128.49	116.64	111.72	114.88	149.94	185.20	194.62
	2.50	133.82	120.69	112.46	108.73	110.07	150.57	193.83	217.22
	3.00	121.05	114.83	108.65	105.71	108.37	150.80	197.39	228.17
	3.50	114.11	110.75	105.55	103.03	107.58	150.90	199.16	234.02
	4.00	110.00	107.93	103.15	100.80	107.15	150.96	200.15	237.45
3.00	**0.05**	162.99	129.59	117.03	111.80	110.90	118.65	140.98	152.61
	0.50	209.90	141.61	124.15	117.64	136.18	126.54	150.13	147.26
	1.00	252.63	148.23	127.58	120.31	1468.70	386.59	181.86	138.89
	1.50	209.90	141.61	124.15	117.64	136.18	126.54	150.13	147.26
	2.00	159.16	128.23	116.14	111.05	109.94	118.34	140.53	152.94
	2.50	132.81	116.28	107.38	103.19	104.97	116.77	138.08	154.96
	3.00	119.70	107.83	100.00	95.97	103.22	116.21	137.15	155.81
	3.50	112.61	102.20	94.38	90.07	102.41	115.95	136.71	156.24
	4.00	108.43	98.43	90.24	85.47	101.97	115.81	136.46	156.48

Table 6. ***PRE*** **of** $\hat{T}_{(\delta,\alpha,p,q)}$ **with respect to** $\hat{Q}^{*}_{(\delta,\alpha,p)}$ **for** α **=-1**

(Applicable for estimation of Process capability index C_p**)**

q	λ	p=1				p=2			
		5	10	15	20	5	10	15	20
1.50	**0.05**	117.66	106.08	103.62	102.58	238.12	215.09	171.59	151.72
	0.50	112.86	104.18	102.48	101.76	135.40	125.87	115.26	110.79
	1.00	110.81	103.42	102.03	101.45	68.12	84.80	91.45	94.04
	1.50	112.86	104.18	102.48	101.76	135.40	125.87	115.26	110.79
	2.00	118.32	106.36	103.79	102.70	248.40	226.59	179.36	157.51
	2.50	125.64	109.74	105.88	104.21	329.79	344.55	266.80	225.27
	3.00	133.31	113.98	108.62	106.23	378.89	451.24	360.46	303.53
	3.50	140.40	118.75	111.86	108.68	408.45	537.25	448.80	383.40
	4.00	146.50	123.74	115.44	111.45	426.99	603.38	526.39	458.98
2.00	**0.05**	120.19	106.86	104.07	102.89	205.37	201.12	165.91	148.44
	0.50	121.01	106.81	104.05	102.88	150.72	131.30	118.15	112.73
	1.00	121.39	106.79	104.04	102.87	93.65	93.43	96.19	97.28
	1.50	121.01	106.81	104.05	102.88	150.72	131.30	118.15	112.73
	2.00	120.08	106.87	104.07	102.90	209.51	209.09	172.08	153.27
	2.50	118.97	106.96	104.12	102.92	236.83	280.07	235.40	206.14
	3.00	117.96	107.06	104.18	102.96	249.71	331.13	292.83	259.87
	3.50	117.14	107.17	104.24	102.99	256.49	365.59	339.24	308.01
	4.00	116.50	107.27	104.30	103.04	260.43	388.86	374.98	348.40
2.50	**0.05**	120.80	106.98	104.11	102.91	172.72	182.27	157.25	143.18
	0.50	126.79	108.96	105.38	103.85	155.87	134.39	120.09	114.10
	1.00	129.69	109.78	105.89	104.22	127.41	102.84	101.14	100.61
	1.50	126.79	108.96	105.38	103.85	155.87	134.39	120.09	114.10
	2.00	120.07	106.71	103.93	102.77	173.74	187.04	161.68	146.90
	2.50	112.86	103.57	101.77	101.12	179.84	224.71	203.15	184.79
	3.00	106.90	100.11	99.19	99.07	182.37	247.34	235.31	218.58
	3.50	102.42	96.76	96.45	96.79	183.63	260.92	258.23	245.50
	4.00	99.17	93.74	93.76	94.44	184.34	269.43	274.30	266.00
3.00	**0.05**	120.70	106.72	103.87	102.70	148.99	163.35	147.19	136.69
	0.50	130.27	110.60	106.47	104.67	154.04	135.20	121.06	114.89
	1.00	135.12	112.26	107.54	105.46	166.99	112.55	106.12	103.94
	1.50	130.27	110.60	106.47	104.67	154.04	135.20	121.06	114.89
	2.00	119.56	106.19	103.50	102.42	148.73	165.82	150.06	139.32
	2.50	108.79	100.32	99.23	99.06	147.23	183.61	174.73	164.22
	3.00	100.37	94.20	94.36	95.04	146.64	193.01	191.51	183.93
	3.50	94.34	88.60	89.45	90.77	146.36	198.26	202.41	198.20
	4.00	90.10	83.80	84.86	86.56	146.20	201.41	209.58	208.31

Table 7. ***PRE*** **of $\hat{T}_{(\delta,\alpha,p,q)}$ with respect to $\hat{Q}^{*}_{(\delta,\alpha,p)}$ for α =1 (Applicable for estimation of standard deviation and Mean deviation about mean)**

q	λ	p=1				p=2			
		5	10	15	20	5	10	15	20
1.25	**0.05**	110.74	104.58	102.91	102.13	183.21	166.47	149.93	139.64
	0.50	108.19	102.81	101.66	101.18	130.58	114.64	109.97	107.64
	1.00	107.15	102.12	101.18	100.81	98.58	92.17	93.59	94.80
	1.50	108.19	102.81	101.66	101.18	130.58	114.64	109.97	107.64
	2.00	111.10	104.84	103.09	102.27	188.76	173.48	155.58	144.24
	2.50	115.31	108.04	105.40	104.06	234.52	249.99	221.64	199.59
	3.00	120.16	112.20	108.49	106.48	263.88	327.52	297.81	267.09
	3.50	125.08	117.07	112.24	109.46	282.24	396.77	375.40	340.28
	4.00	129.71	122.38	116.49	112.91	294.04	454.71	448.68	413.92
1.50	**0.05**	111.49	105.04	103.22	102.37	169.40	162.97	148.51	138.83
	0.50	110.59	103.97	102.41	101.73	135.24	117.10	111.42	108.65
	1.00	110.22	103.54	102.09	101.48	110.14	95.95	95.72	96.29
	1.50	110.59	103.97	102.41	101.73	135.24	117.10	111.42	108.65
	2.00	111.62	105.19	103.34	102.46	172.58	168.88	153.62	143.09
	2.50	113.03	107.10	104.84	103.66	196.32	229.39	211.06	192.96
	3.00	114.58	109.50	106.80	105.25	209.55	283.98	272.42	250.45
	3.50	116.07	112.21	109.12	107.18	217.16	327.91	330.14	309.00
	4.00	117.40	115.07	111.69	109.37	221.80	361.62	380.71	364.29
1.75	**0.05**	111.86	105.32	103.43	102.53	156.34	157.90	146.25	137.48
	0.50	112.48	104.99	103.09	102.24	137.77	119.13	112.68	109.55
	1.00	112.74	104.86	102.96	102.13	121.48	99.92	97.92	97.81
	1.50	112.48	104.99	103.09	102.24	137.77	119.13	112.68	109.55
	2.00	111.78	105.36	103.47	102.57	157.89	162.62	150.74	141.35
	2.50	110.84	105.93	104.08	103.10	168.68	207.89	199.07	185.19
	3.00	109.87	106.62	104.85	103.79	174.14	244.40	246.71	232.73
	3.50	108.97	107.38	105.74	104.62	177.12	271.20	288.19	278.13
	4.00	108.20	108.15	106.71	105.53	178.88	290.40	322.15	318.47
2.00	**0.05**	112.01	105.45	103.53	102.61	145.53	151.71	143.30	135.66
	0.50	113.90	105.86	103.71	102.71	138.80	120.68	113.72	110.31
	1.00	114.72	106.03	103.78	102.75	131.90	104.03	100.15	99.35
	1.50	113.90	105.86	103.71	102.71	138.80	120.68	113.72	110.31
	2.00	111.75	105.39	103.50	102.60	146.04	155.27	147.12	139.10
	2.50	108.97	104.68	103.18	102.41	149.46	187.28	186.47	176.68
	3.00	106.16	103.84	102.77	102.18	151.09	210.58	222.28	214.93
	3.50	103.65	102.94	102.31	101.90	151.95	226.44	251.31	249.25
	4.00	101.56	102.06	101.83	101.60	152.44	237.20	273.72	278.08

It is observed from Tables 2, 3 and 4 that :

(i) for fixed (n,p,q) percent relative efficiency (PRE) of the proposed class of estimators $\hat{T}_{(\delta,\alpha,p,q)}$ relative to the MMSE estimator $\hat{Q}^{(m)}_{(\delta,\alpha)}$ increase up to $\lambda = 1$ (i.e. when the guessed value exactly coincides with the true value), attains its maximum at this point and then decrease symmetrically in magnitude, as λ goes up in its range of dominance for all (n,p,q) .

(ii) for fixed (λ,p,q) PRE decreases as n increases.

(iii) for fixed (λ, n) with $p=1$, the PRE increases as q increases except when $(q,\lambda)=(3.00,2.00)$ and it is reversed in case with *p=2*. The PRE with *p*=2 is more as compared to *p*=1.

(iv) the proposed estimator $\hat{T}_{(\delta,\alpha,p,q)}$ offers larger gain in efficiency over MMSE estimator $\hat{Q}^{(m)}_{(\delta,\alpha)}$ when λ moves in the neighborhood of 'unity', sample size n is small , *p*=2 and $0.50 \le \lambda \le 1.50$.

Further from Tables 5, 6 and 7, it is observed that for fixed (λ, p, q) PRE decreases as n increases. The proposed class of estimators $\hat{T}_{(\delta,\alpha,p,q)}$ is more efficient than the family of estimators $Q^{*}{}_{(\delta,\alpha,p)}$ with substantial gain in efficiency except in few cases. It is further observed that there is enough scope of choosing scalars (p,q) to obtain better estimators than the MMSE estimator $\hat{Q}^{(m)}_{(\delta,\alpha)}$ and the estimator $Q^{*}{}_{(\delta,\alpha,p)}$ from the suggested class of estimators $\hat{T}_{(\delta,\alpha,p,q)}$.

V. CONCLUSION

It is interesting to mention that the proposed class of estimators $\hat{T}_{(\delta,\alpha,p,q)}$ estimates numerous important parameters. The proposed class of estimators includes different classes of estimators of various parameters whose biases and mean squared errors can be obtained easily. The suggested class of estimators has considerable gain in efficiency for a number of choices of scalars involved in it, particularly for small sample sizes. Such sample sizes are desirable when the cost per unit is very high.

Lastly we conclude that even if the experimenter has little experience in the guessed value σ_0^2 of σ^2, the efficiency of the proposed class of estimators can be increased substantially by selecting the scalars p and q. The suggested class of shrinkage estimators is therefore recommended for its use in practice.

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